CENTRAL LIMIT THEOREM

- specifies a theoretical distribution
- formulated by the selection of all possible random samples of a fixed size n
- a sample mean is calculated for each sample and the distribution of sample means is considered

SAMPLING DISTRIBUTION OF THE MEAN

- The mean of the sample means is equal to the mean of the population from which the samples were drawn.
- The variance of the distribution is σ divided by the square root of n. (the standard error.)

STANDARD ERROR

Standard Deviation of the Sampling Distribution of Means

$\sigma_x = \sigma / \sqrt{n}$

How Large is Large?

- If the sample is **normal**, then the sampling distribution of $\bar{x}$ will also be normal, no matter what the sample size.
- When the sample population is approximately **symmetric**, the distribution becomes approximately normal for relatively small values of *n*.
- When the sample population is **skewed**, the sample size must be **at least 30** before the sampling distribution of $\bar{x}$ becomes approximately normal.

EXAMPLE

A certain brand of tires has a mean life of 25,000 miles with a standard deviation of 1600 miles.

What is the probability that the mean life of 64 tires is less than 24,600 miles?

Example continued

The sampling distribution of the means has a mean of 25,000 miles (the population mean)

μ = 25000 mi.

and a standard deviation (i.e.. standard error) of:

1600/8 = 200

Example continued

Convert 24,600 mi. to a z-score and use the normal table to determine the required probability.

$z = (24600-25000)/200 = -2$

$P(z < -2) = 0.0228$

or 2.28% of the sample means will be less than 24,600 mi.

ESTIMATION OF POPULATION VALUES

- Point Estimates
- Interval Estimates

CONFIDENCE INTERVAL ESTIMATES for LARGE SAMPLES

- The sample has been randomly selected
- The population standard deviation is known or the sample size is at least 25.

Confidence Interval Estimate of the Population Mean

$$\bar{X} - z\frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + z\frac{s}{\sqrt{n}}$$

$\bar{X}$: sample mean

s: sample standard deviation

n: sample size

EXAMPLE

Estimate, with 95% confidence, the lifetime of nine volt batteries using a randomly selected sample where:

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X = 49 hours

s = 4 hours

n = 36

EXAMPLE continued

Lower Limit: 49 - (1.96)(4/6)

49 - (1.3) = 47.7 hrs

Upper Limit: 49 + (1.96)(4/6)

49 + (1.3) = 50.3 hrs

We are 95% confident that the mean lifetime of the population of batteries is between 47.7 and 50.3 hours.

CONFIDENCE BOUNDS

- Provides a upper or lower bound for the population mean.
- To find a 90% confidence bound, use the z value for a 80% CI estimate.

Example

- The specifications for a certain kind of ribbon call for a mean breaking strength of 180 lbs. If five pieces of the ribbon have a mean breaking strength of 169.5 lbs with a standard deviation of 5.7 lbs, test to see if the ribbon meets specifications.
- Find a 95% confidence interval estimate for the mean breaking strength.